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(20222)

M.A./M.Sc.-I Sem.

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# 11092 CV–III

## M.A./M.Sc. Examination, December-2021

## METRIC SPACES

## (GH-1052)

*Time : 1½ Hours]* *[Maximum Marks : 50*

**Note :** Attempt questions from each sections as per instructions.



### Section–A

### (Very Short Answer Questions)

*Note :* Attempt any **two** questions of this section. Each question carries 5 marks. Very short answer is required.

$2 \times 5 = 10$

1. Consider $R^2$ and let $\rho$ be a real valued function defined on $R^2 \times R^2$ by setting $\rho(x, y) = \min\{|x_1 - y_1|, |x_2 - y_2|\}$ where $x = (x_1, x_2)$, $y = (y_1, y_2)$. Is $\rho$ a metric ?

2. In a discrete metric space (X, d), prove that every Singleton is an open set.





3. Find the closure of Q the set of rational numbers in (R, d) where R is the set of real numbers equipped with the standard metric.

4. Let (X, d) and (Y, $\rho$) be metric spaces and let $T : (X, d) \rightarrow (Y, \rho)$ be a contraction mapping. Show that T is continuous.

5. Give an example of a subspace of a metric space (X, d) which is complete but not compact.

### Section–B

### (Short Answer Questions)

*Note :* This section contains **three** questions. Attempt any one question. Each question carries ten marks. Very short answer is required. $1 \times 10 = 10$

6. Prove that every complete subspace of a metric space is closed.

7. Prove that the continuous image of a connected metric space is connected.

8. Let (X, d) be a metric space and let A and B be any subsets of X. Show that :

(i) $A^\circ$ [the interior of A] is the largest open subset of A



(ii) $A \subseteq B \Rightarrow A^\circ \subseteq B^\circ$.

(iii) $(A \cup B)^\circ = A^\circ \cup B^\circ$.



## Section–C

**(Detailed Answer Questions)**

*Note :* This section contains five questions. Attempt any two questions. Each question carries 15 marks. Answer is required in detail. 2×15=30

9. Let (X, d) be a metric space and let ρ be a real valued function on R × R defined as $\rho(x, y) = \min\{1, d(x, y)\}\ \forall x, y \in X$. Show that ρ is a metric on X.





10. Let $\{G_1, G_2, \ldots\ldots, G_n\}$ be a finite collection of open subsets of a metric space (X, d). Then prove that $\bigcup_{i=1}^{n} G_i$ is an open subset of X, What can you say about $\bigcup_{i=1}^{n} G_i$ ? Justify your answer.

11. Let $(X_1, d_1)$ and $(X_2, d_2)$ be metric spaces and let $f : X_1 \rightarrow X_2$ be a mapping from $X_1$ into $X_2$. Show that f is continuous if and only if $f^{-1}(G)$ is an open subset of $X_1$ whenever G is an open subset of $X_2$.

12. Prove that every contraction mapping defined on a complete metric space (X, d) has a unique fixed point.

13. Let (R, d) be a metric space where d is the standard metric defined on the set of Real numbers R. Show that a subset S of (R, d) is compact if and only if it is closed and bounded.